Mass Transfer in a Horizontal Rotating Cylinder with Applications to the Oxygenation of Blood

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The horizontal rotating cylinder blood oxygenator has been developed to minimize shear, eliminate bubbles, reduce holdup, and provide for rapid change of the controlled temperature of the blood.

To aid in the development of this apparatus a detailed mathematical model was formulated. A simplified model was also developed which yielded an analytical expression relating the amount of material transferred between the gas and liquid in the apparatus to operating variables. In developing these models an attempt was made to include a realistic mechanism for oxygen absorption in blood.

Preliminary experiments were carried out by absorbing carbon dioxide into distilled water in the horizontal rotating cylinder. These preliminary results were found to be in good agreement with previous theoretical predictions and calculations that indicated that the equipment should be adequate to act as a lung substitute for a dog. When it was employed in this manner during open-heart surgery, insufficient oxygen was transferred to support the dog.

It is concluded that the most probable reason for failure of the horizontal cylinder to act satisfactorily as a blood oxygenator is poor mixing in the blood pool.

Manmade substitutes for various parts of the body are finding increasing applications. These include substitutes which remain as a permanent part of the living organisms, such as plastic arteries and heart valves as well as a variety of removable attachments. Such attachments are of two types: those which do not form a part of the living organism, such as limbs and the artificial lung; and temporary substitutes for living organs, such as the heart, kidney, and the lung (blood oxygenator). The artificial lung is primarily a mechanical device which permits a person to utilize his own lungs by taking over the normal actions of the thorax muscles. On the other hand, the blood oxygenator provides for mass transfer of oxygen to and carbon dioxide from the blood and actually takes over the normal functions of the lung itself. Blood oxygenators are not generally used as replacements for diseased or faulty lungs but usually are used to simplify open-heart surgery.

In performing any one of a number of operations on the heart, it is essential to isolate this organ from the circulatory system to provide increased visibility, reduce loss of blood, and increase the allowable time for operating. Naturally, it is essential that the flow of blood be continued through the remaining parts of the body during such isolation. Substitution of a pump for the heart is somewhat complicated by the fact that the heart acts as a two-stage pump, as illustrated in Figure 1. The first stage (right atrium and right ventricle) pumps the blood through the lungs where it is oxygenated and the second stage (left atrium and left ventricle) then provides pumping action to force the blood through the other body organs. Instead of providing a two-stage mechanical heart and of using the patient's lungs, it is found convenient to use an oxygenator with low pressure drop and a single heart pump as shown in Figure 1.

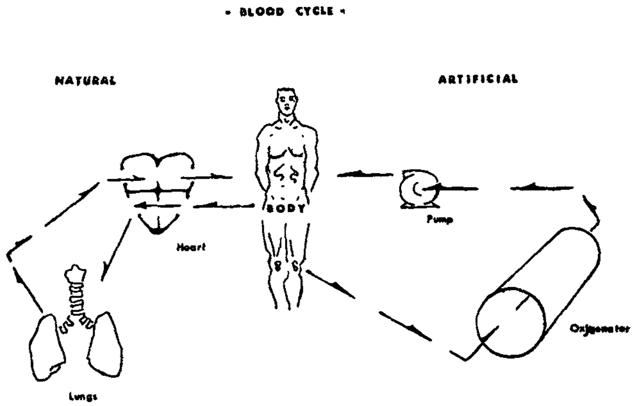

Fig. 1. Natural and artificial blood cycles.

Although the blood oxygenator acts primarily as a mass transfer device, there are several special requirements which must be satisfied by blood oxygenators. The blood itself contains red corpuscles which are filled with hemoglobin in stroma. If these cells are ruptured the release of the stroma and hemoglobin into the blood plasma has harmful effects. Therefore, blood oxygenators (as well as any device which handles blood) must be designed to minimize trauma and shear. If bubbles are present in blood in the human body, they can cause blockage of the flow resulting in permanent damage. Foaming or any other source of bubbles in blood must be prevented or a debubbling device must be provided to insure that no bubbles enter the patient's body. Formation of blood clots in the oxygenator would cause similar difficulties. Anticoagulants are used to minimize the tendency for clot formation during open-heart surgery and blood oxygenators should be designed so that no blood remains in the oxygenator for excessive periods of time. Relatively little blood can be removed from the patient's circulatory system for the

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purpose of filling the blood oxygenator. As a result it is generally necessary to obtain whole blood from donors to prime the oxygenator. Mismatch of blood, although uncommon, can be fatal. For these reasons it is desirable to keep the holdup of blood in the oxygenator and auxiliary equipment as small as possible. This factor is of special importance in the case of open-heart surgery on infants. The nature of the direct application of a blood oxygenator dictates that it must be reliable and, in addition, it is desirable that the cost of the equipment be held to a reasonable amount.

It has been determined that open-heart surgery can be performed successfully for longer periods of time when the blood is introduced into the body at lower than normal temperatures. Such temperature control can be provided apart from the oxygenator, but incorporation of such control as part of the bood oxygenator would reduce total blood holdup.

Several types of oxygenators have been used. Perhaps the most popular is the so-called *bubbler oxygenator* in which pure oxygen is bubbled through countercurrent flowing blood in a small chamber. The mass transfer characteristics of such a device are very good. Trauma and shear experienced in the bubbles, while not excessive, are severe enough to limit the use of such units to periods of several hours or less. The blood leaving the bubbler contains foam and bubbles and it is necessary to provide a debubbling device. Many of the disadvantages of early debubblers such as very large holdup have been corrected.

The design of several other oxygenators is based on the principle of providing an extended surface of blood for contact with the oxygen. The *screen oxygenator* consists of a very fine vertical screen over which blood is poured and through which oxygen is blown. The blood tends to stagnate at the intersection of two wires and may form clots. Not only does this represent a potential danger to the patient, but it also makes the screens difficult to clean.

The *disk oxygenator* consists of a series of metal disks mounted on a horizontal rotating shaft. The disks are partially submerged in a pool of blood. As the disks rotate they lift a film of blood from the pool, thereby providing extended surface for contact with the oxygen atmosphere which surrounds the remainder of the disks. In order to provide proper submergence of the disks considerable holdup of blood is required. In addition, blood tends to run down the disks onto the rotating shaft and remain there.

Although the bubbler oxygenator has been used in many successful operations, none of the oxygenators in current use are entirely satisfactory. An oxygenator with very small holdup of blood would represent a significant advancement, especially in the field of application to infants. The horizontal rotating cylinder oxygenator developed by Garman Kimmell and his associates at Kimray, Inc., Oklahoma City, is an attempt to fulfill this need. The purpose of the investigations described here was to develop suitable mathematical models and to carry out laboratory tests on the unit as a means of guiding the research effort and reducing development costs.

THE HORIZONTAL ROTATING CYLINDER OXYGENATOR

A schematic diagram of this oxygenator is presented in Figure 2. As indicated in the figure, blood flows countercurrent to oxygen through one cylindrical section into a second cylindrical section of larger diameter. The increase in diameter of the cylinder is made to provide a sump for collection of the oxygenated blood and its removal from the oxygenator. Most of the mass transfer occurs in the cylindrical section of smaller diameter. This section contains a pool of blood which flows along the bottom of the cylinder and a film which is carried out of the pool,

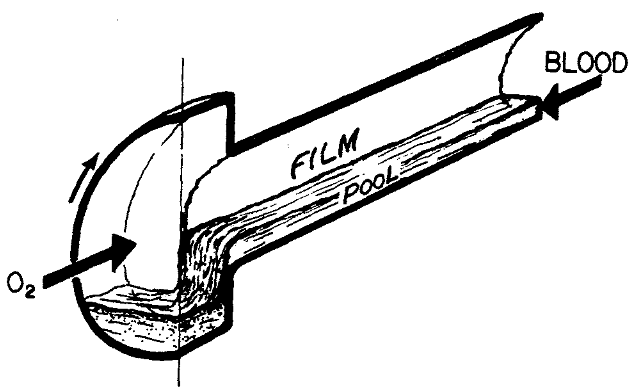

Fig. 2. Schematic diagram of the horizontal rotating cylinder oxygenator.

is contacted with the oxygen, and is returned to the pool by the rotation of the cylinder. The primary functions of the pool are to provide for bulk transport of the blood from one end of the oxygenator to the other, to supply blood to the film, and to mix the oxygenated blood back into the pool as the film reenters. The film provides an extended surface for transfer of oxygen and carbon dioxide.

The horizontal rotating cylinder meets the special requirements of blood oxygenators very well. No bubbles appear in the blood during normal operation. The blood experiences very moderate conditions and shear is minimum. Proper design insures that very little blood is required to prime the apparatus and the residence time is low with no "dead spots" so that there is little danger of coagulation. This design has another desirable feature in that the temperature of the blood can be controlled by spraying fluid at the proper temperature on the outside of the cylinder. There is very limited experience in the operation of the horizontal rotating cylinder oxygenator, but the simplicity of design should be a contributing factor to high reliability and reasonable cost.

THEORY

The objectives of the theoretical analysis are to relate the mass transfer in the oxygenator to its dimensions and operating variables. In the analysis that follows attention will be restricted to the section of the cylinder of small diameter in which the majority of the mass transfer occurs.

A rigorous analysis was made in an attempt to develop a better understanding of the hydrodynamics as they influence the mass transfer in the horizontal rotating cylinder oxygenator (*6*). This analysis led to a set of nonlinear partial differential equations which were not amenable to analytical solution. Therefore, several simplifications were incorporated in the development to obtain a comparatively simple expression to guide the experimental investigations.

In making this simplified approach it was assumed that the pool acted only to provide for bulk flow of blood through the oxygenator, to provide blood to the film, and to accept oxygenated blood from the film. It was further assumed that all transfer of oxygen from the gas phase to the blood occurred solely in the film and that no blood was carried through the oxygenator by the film.

The hydrodynamics of the pool were simplified by assuming that the blood flows at uniform velocity (rodlike flow) in the direction of the axis of the cylinder (z coordinate). A one-dimensional model of the pool was obtained by assuming perfect mixing in the pool in the plane perpendicular to the axis. A steady state material balance was made, taking into account: (1) the oxygen carried into and out of a differential element of the pool by the bulk flow of blood with concentration of total available oxygen, $C(z)$ (expressed as volume percent); (2) the oxygen leaving the pool in the film with concentration also equal to $C(z)$ as a result of assuming perfect mixing; (3) the oxygen entering the pool in the oxygenated film at concentration $C^*(z)$; and neglecting (1) diffusion of oxygen in the z direction; (2) absorption of oxygen through the surface of the pool; (3) any change in the volumetric flow of blood M; and (4) any difference in thickness t of the blood film leaving and entering the pool.

The following linear first-order, separable differential equation results:

$$M \frac{dC(z)}{dz} = \omega Rt \, [C^*(z) - C(z)] \qquad (1)$$

To integrate this equation and to determine the manner in which the oxygen content of the blood varies along the length of the oxygenator, it is necessary to obtain a relation between the oxygen content of the film reentering the pool $C^*(z)$ and the concentration of the blood in the mixed pool at any point z, $C(z)$.

A limiting solution is obtained by assuming that the film reentering the pool is completely oxygenated, that is, $C^*(z) = C_s$. Substitution of this simple relation into Equation (1) and integration yields a limiting relation between the oxygen concentration entering the oxygenator C_i and leaving it C_o in relation to the oxygen content of blood which is saturated with respect to the partial pressure of oxygen in the oxygenator C_s.

$$\frac{C_o - C_i}{C_s - C_i} = 1 - \exp\left(-\frac{\omega RtL}{M}\right) \qquad (2)$$

Under most conditions of operation, the film reentering the pool will not be saturated and it is necessary to give detailed consideration to the accumulation of total available oxygen in the film. In developing a partial differential equation which is descriptive of the accumulation of total available oxygen in the film, we simplified the hydrodynamics of the film by assuming it to be uniform in thickness and to move at a velocity equal to the peripheral velocity of the cylinder. Further simplification was obtained by ignoring the cylindrical nature of the film and by treating it in terms of pseudo Cartesian coordinates, a simplification justified by the fact that the thickness of the film is very much smaller than the radius of the cylinder. The red blood cells do not move in the film (*1*) and it is reasonable to ignore oxygen transfer by diffusion in the direction of movement (rotation) of the film. With these assumptions oxygen transfer in the direction perpendicular to the surface of the film (y coordinate) results solely from diffusion, whereas oxygen transfer in the direction of the movement of blood (x coordinate) results from bulk movement of the total available oxygen in the film. Considering that the rate of diffusion of oxygen is proportional to the gradient of physically dissolved oxygen in the plasma $\partial C'(x, y)/\partial y$, we obtained the following partial differential equation:

$$D \frac{\partial^2 C'(x, y)}{\partial y^2} = \omega R \frac{\partial C(x, y)}{\partial x} \qquad (3)$$

To solve this equation it is necessary to relate the concentration of oxygen dissolved in the plasma $C'(x, y)$ and the total available oxygen in the blood $C(x, y)$. In general, this requires detailed consideration of the mechanisms by which blood is oxygenated: diffusion through the plasma to the red corpuscle, transfer through the membrane of the red corpuscle, diffusion through the stroma within the red blood cell, and chemical combination of oxygen and hemoglobin.

Unsteady state diffusion of oxygen into relatively thick films of whole blood (such as would be encountered in the horizontal rotating cylinder oxygenator) has not been the subject of extensive theoretical or experimental investigations. Fatt and LaForce (*2*) considered steady state diffusion of oxygen through whole blood (in which case chemical reaction plays no role) and found that the presence of red blood cells causes a decrease in the rate of mass transfer by diffusion in comparison to diffusion through plasma alone. This effect should be taken into account when assigning a numerical value to the effective diffusion coefficient D in Equation (3).

Roughton (*10*) has given detailed consideration to the last three mechanisms involving transfer into and through the red blood cell and chemical reaction with hemoglobin within the cell. By using techniques designed for the determination of rapid reaction velocities and by working both with homogeneous solutions of hemoglobin and red cell suspensions, it was established that the rate of chemical reaction is relatively rapid in comparison with the two diffusive transfer processes. The results of theoretical calculations indicate that the resistances to oxygen transfer of the membrane and of the stroma inside the cell are approximately equal. The sum total result of these three factors is such that human red blood cells achieve 50% of saturation with respect to the surrounding medium in about 0.1 sec.

Preliminary estimates made on the basis of the data presented by Roughton and a measured value of the blood film thickness in the horizontal rotating cylinder ($\simeq$ 0.015 cm.) indicate that neither diffusion through the plasma nor the net result of the three steps involved in the "reaction" of the red blood cell with the dissolved oxygen would be rate limiting under conditions of operation of the horizontal rotating cylinder. However, it appears that in the relatively thick film of the oxygenator, diffusion will probably be the more important of the two factors. Therefore, to obtain a solution of reasonable mathematical form, it is assumed that diffusion of oxygen through the blood is the rate-controlling step, that is, it is assumed that the "reaction" between the red blood cells and the dissolved oxygen is instantaneous. Under these conditions local equilibrium exists between the oxygen in the plasma and the oxygen in the red cell.

Making use of this model, it is possible for one to relate the concentration of oxygen dissolved in the plasma $C'(x, y)$ and the total available oxygen in the blood $C(x, y)$, using data obtained under equilibrium conditions.

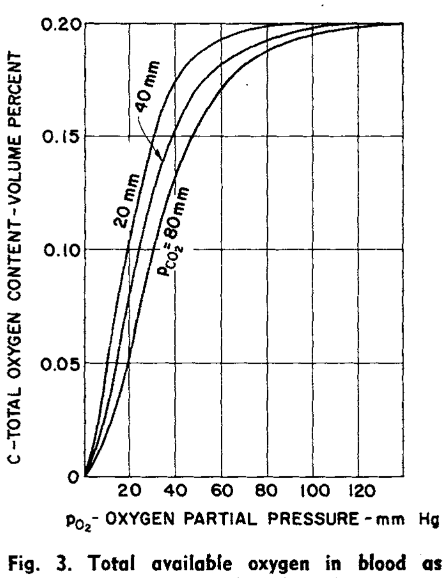

Fig. 3. Total available oxygen in blood as functions of oxygen and carbon dioxide partial pressures.

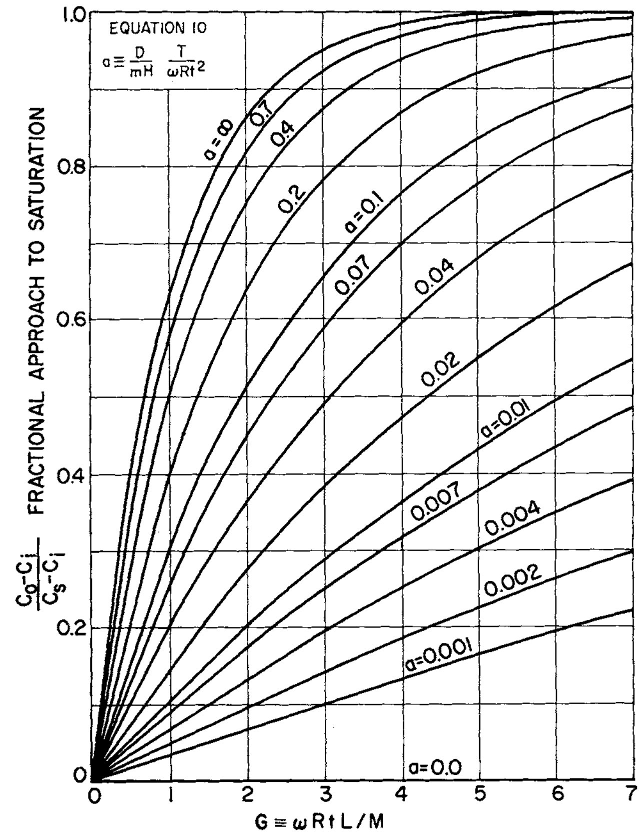

Fig. 4. Fractional approach to saturation as predicted by Equation (10) (0<G<7).

For the case of physical solution of oxygen in blood plasma (3) the data are well represented by Henry's law.

$$p_{O_2} = HC' \tag{4}$$

The relation between the total available oxygen in the blood C and the partial pressure of oxygen (4) is somewhat more complicated, as illustrated in Figure 3. In order to use these data directly, one would require a numerical solution. An approximation is employed which permits one to obtain an analytical solution to the problem.

Thews (*11*) applied a very similar model to describe the transfer of oxygen through homogeneous hemoglobin solutions. Thews approximated the data presented in Figure 3 by means of a linear equation that passes through the origin.

$$C = N\ p_{O_2} \tag{5}$$

Such a relation provides a reasonable fit of the data at relatively low-oxygen partial pressures. In the operation of a blood oxygenator with oxygen partial pressures up to 760 mm., the approximation suggested by Thews is of questionable applicability. However, venous blood returned to the oxygenator under normal conditions should contain appreciable amounts of oxygen, so that a slightly different linear form should provide a reasonable fit to the data over a considerable range of higher oxygen partial pressures.

$$C = m\,p_{O_2} + j \tag{6}$$

Simultaneously with the transfer of the oxygen into the blood, carbon dioxide is transferred out of the blood into the stream of oxygen. The mechanism of carbon dioxide desorption is essentially the reverse of the mechanism of oxygen absorption, one important difference being that most of the carbon dioxide remains in the plasma rather than existing in chemical combination in the red cells. The blood oxygenator is continually flushed with pure oxygen and therefore the effect of the carbon dioxide "desorption" will be considered only as its presence affects the equilibrium oxygen content of blood. Figure 3 illustrates this effect. The carbon dioxide partial pressure in venous blood is 46 mm. Hg, whereas arterial blood leaving the lungs is at about 40 mm. Hg. For the present study the oxygen-hemoglobin equilibrium curve at 40 mm. Hg partial pressure of carbon dioxide was approximated by Equation (6).

Both Equations (5) and (6) are linear and therefore the results obtained by Thews (*11*) can be applied directly with minor modification of symbols. Thus solution of Equations (3), (4), and (6) subject to suitable boundary conditions as applied by Thews yields the desired relation between the oxygen content of the blood in the film entering the pool at point z, $C^*(z)$, and the blood in the pool at point z, $C(z)$.

$$C^*(z) = C_s + b[C(z) - C_s] \tag{7}$$

where

$$b = \frac{8}{\pi^2}\sum_{n=0}^{\infty}\frac{\exp\left[-(2n+1)^2\frac{\pi^2}{4}a\right]}{(2n+1)^2} \tag{8}$$

and

$$a \equiv \frac{D}{mH}\,\frac{T}{\omega Rt^2} \tag{9}$$

Equation (7) serves to account for the factors which influence the uptake of oxygen in the film, and when substituted into Equation (1) and integrated, yields the desired equation relating the oxygen content entering the oxygenator C_i and leaving it C_o to the dimensions and operating parameters of the oxygenator.

$$\frac{C_o - C_i}{C_s - C_i} = 1 - \exp\left[-G\,(1-b)\right] \tag{10}$$

where

$$G \equiv \frac{\omega RtL}{M} \tag{11}$$

This expression satisfies the primary purpose of the theoretical analysis and expresses the performance of the oxygenator in terms of the two dimensionless groups, a and G. Equation (10) is presented in graphical form in Figures 4 and 5. The former applies for low values of G and the latter applies for values of G up to 100.

These graphs are useful in predicting the effect on the total oxygen absorption of changes in cylinder dimensions

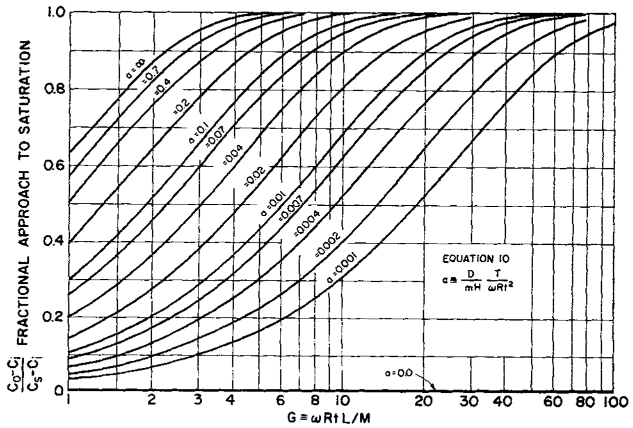

Fig. 5. Fractional approach to saturation as predicted by Equation (10) (1<G<100).

TABLE 1. EXPERIMENTAL RESULTS WITH THE HORIZONTAL ROTATING CYLINDER OXYGENATOR

Run No.	rev./min.	Cylinder length (L), cm.	Flow rate (M), cc./min.	Parameter L/M, min./sq. cm.	Outlet normality, C_o	Saturation normality, C_s	Fractional approach to saturation C_o/C_s Exptl.	Pred.
1	20	100	423.5	0.236	0.0545	0.0594	0.92	0.97
2	20	100	1534	0.0653	0.0304	0.0585	0.52	0.58
3	20	100	1062	0.0942	0.0324	0.0555	0.60	0.745
4	20	85	1330	0.064	0.0324	0.0676	0.48	0.565
5	12.3	85	2105	0.0402	0.0164	0.0585	0.2805	0.395
6	12.3	85	585	0.1452	0.0415	0.0668	0.621	0.82
7	20	100	680	0.1470	0.0472	0.0670	0.705	0.885
8	20	100	660	0.150	0.0525	0.0658	0.798	0.890
9	12.3	100	941	0.106	0.0365	0.0668	0.547	0.715
10	12.3	100	508	0.197	0.0432	0.0618	0.699	0.895
11	30	100	830	0.1204	0.0554	0.0649	0.854	0.85
12	30	100	630	0.1588	0.0506	0.0571	0.886	0.91

and operating parameters. The physical properties of the system are represented by the group (D/mH), which appears only in the dimensionless group a. Therefore, an increase in the value of the group (D/mH), with all other factors being held constant, will cause a nearer approach to equilibrium in the oxygenator. The length of the oxygenator L and the mass flow rate of blood through it M appear as the ratio L/M in the G group only. Therefore, any increase in the ratio L/M will result in a closer approach to equilibrium in the oxygenator. Under most circumstances the film length T will be proportional to the cylinder radius R and therefore the dimensionless group a will be independent of the dimensions of the oxygenator. As a result, the influence of increasing the radius of the cylinder will be determined uniquely by a proportional change in G. Not only do the angular velocity ω and the film thickness t both appear in each of the dimensionless groups a and G, but they are undoubtedly hydrodynamically related. Therefore, the influence of changes in these two variables must be investigated for special cases.

The mathematical analysis of the oxygenator was simplified by ignoring the oxygen transfer directly to the pool and by ignoring the section of increased diameter that acted as the sump for the oxygenated blood. These omissions were partially corrected for by taking the film length T to be equal to the entire circumference of the cylinder and by calculating the oxygenator length L as the sum of the lengths of both the small-diameter and large-diameter sections.

EXPERIMENTS

The numerous assumptions made in the development of the mathematical analysis of the horizontal rotating cylinder oxygenator, although reasonable, necessitated experimental investigation. It seemed desirable to carry out experiments using blood and oxygen, but difficulties associated with providing blood for continuous flow through the oxygenator, and the fact that methods of analysis for total oxygen content of blood are both complicated and not highly accurate, led to the decision to carry out a preliminary investigation with another physical system.

It was decided to investigate the dissolution of carbon dioxide into water in the horizontal rotating cylinder. This choice was largely a matter of convenience because these reagents are readily available in reasonable purity, and analysis by titration provides a rapid and accurate method of determining the carbon dioxide content of distilled water. Unfortunately, this system is not directly analogous to the oxygen-blood system for several reasons:

1. Although carbon dioxide reacts to some extent in water, at ordinary temperatures and pressures the majority of the carbon dioxide is in physical solution rather than chemical combination (5), whereas the opposite is true for oxygen in blood.

2. Water is a Newtonian fluid, whereas blood has non-Newtonian characteristics.

In spite of these differences, results obtained with the carbon dioxide–water system serve to test several of the assumptions used in developing the mathematical model, including those of ideal mixing and rodlike flow in the pool and uniformity of thickness of the film. It was felt that application of the results of the theoretical analysis to predict the performance of the equipment with oxygen and blood would be justified in part if these important assumptions were substantiated by experiments with the carbon dioxide–water system.

Equipment

Figure 2 is a schematic diagram of the horizontal rotating cylinder. The stainless steel cylinder which forms a major part of the contactor is 16 cm. in radius and has a total length of 1 meter. The cylinder is supported on a hollow shaft through which the liquid and gas are introduced and removed. The length of the sump section is 16 cm. and it has a radius of 22 cm.

Procedure

The variables required to establish the performance of the contactor and to provide for a theoretical check of the results were either obtained by direct measurement or by use of data from the literature. In starting an experiment the rotation rate ω was set at a steady value and measured by determining the length of time required for ten revolutions. The film thickness was measured by determining experimentally the volume of the water in the film. To accomplish this, the cylinder was filled with water to a well-defined mark with water being recirculated externally from the outlet to the inlet. Rotation was started and as the water filmed onto the walls the level in the sump went down. Water was added to bring the level back to the mark and the amount noted. Knowing the area of the film, one could easily calculate the film thickness. The film thickness was found to be independent of rotation rate within the limits of accuracy of this method.

The volume flow rate of liquid M was measured by determining the time necessary to fill a liter-graduated cylinder. During the first several runs the flow rates into and out of the cylinder were adjusted manually, but on later runs an automatic control device was used to maintain a constant holdup of liquid in the cylinder. The supply was a siphon system of several 5-gal. bottles with a head of about 6 ft. which varied slightly as the supply diminished. The length of the horizontal cylinder could be varied by inserting a doughnut shaped disk with surgical rubber gasket which served as a dam at the point where the water entered. The water was entered downstream from the dam through a tube.

The carbon dioxide content of the distilled water was negligible. The pH of this water was tested periodically. The concentration of carbon dioxide in the outlet stream was determined by titration. Samples were obtained by opening a side line and flowing outlet solution through a 25-cc. pipette.

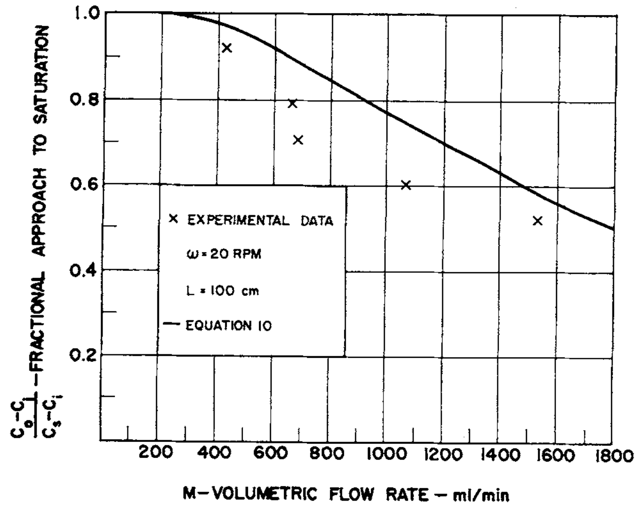

Fig. 6. Effect of volumetric flow rate of liquid on the approach to saturation.

The contents of the pipette were then drained into 25 cc. of barium hydroxide in excess to precipitate all the carbon dioxide as barium carbonate. The excess barium hydroxide was titrated with hydrochloric acid to the phenolphthalein end point.

A thermometer was placed inside the sump to indicate the temperature of the liquid within the cylinder and it was assumed that the partial pressure of carbon dioxide plus the vapor pressure of water was 760 mm. Hg.

Results

The results of the experimental determinations made with carbon dioxide and water in the horizontal rotating cylinder oxygenator are presented in Table 1 (*7*). During the course of this investigation the rotation rate was varied from 12.3 to 30 rev./min., the cylinder length from 85 to 100 cm., and the flow rate of water from 400 to 2,100 cc./min. The reported values of the fractional approach to saturation calculated using available data on the equilibrium concentration of carbon dioxide in water (*8*) were found to vary from 0.28 to 0.92.

COMPARISON OF THEORY WITH RESULTS FROM EXPERIMENTS WITH THE CARBON DIOXIDE-WATER SYSTEM

As mentioned previously the results obtained with the carbon dioxide–water system provide a preliminary check on theoretical predictions of the influence of several important parameters on the operation of the horizontal rotating cylinder as a contacting device.

Volumetric Flow Rate of Liquid, *M*

Runs 1, 2, 3, 7, and 8 as presented in Table 1 were obtained at constant values of ω = 20 rev./min. and L = 100 cm. for various values of water flow rates M. These data are presented as X's in Figure 6. Experimentally determined values of operating parameters were used together with values of the diffusion coefficient of carbon dioxide in water from the literature (*9*) to predict the performance of the equipment using the theoretical results presented in Figures 4 and 5. The results of these theoretical calculations are shown as a solid line in Figure 6. There is good qualitative agreement between the predicted and observed influence of volumetric flow rate, but in all cases the theory predicts a closer approach to equilibrium than was actually obtained.

Length of Cylinder, *L*

As mentioned previously a dam was inserted in the cylinder during several experiments to reduce the effective length of the cylinder L from 100 to 85 cm.

The theory predicts that the combined influence of cylinder length L and volumetric flow rate M is represented by the ratio L/M. Runs 2 and 4 were carried out with different lengths and at different flow rates arranged such that the grouping L/M and all other variables were approximately constant. As can be seen by data presented in Table 1, the approach to equilibrium in the two cases was approximately equal.

Runs 5, 6, 9, and 10 were carried out at a constant rotation rate of 12.3 rev./min. for two different lengths. These data are plotted as a function of the ratio L/M in Figure 7. The fact that a smooth curve results gives support to the theoretically predicted dependence on cylinder length and volumetric flow rate. Such plotting also provides for a direct comparison of the experimental results with theoretical predictions made with Figure 5. The solid line in Figure 7 represents theoretical calculations with data from the literature. It was noted that all the experimental data fell below the theoretical curve and could be represented quantitatively as a constant fraction of the theoretically predicted curve. The dashed line in Figure 7 was calculated from the theoretical curve with a constant factor of 0.76.

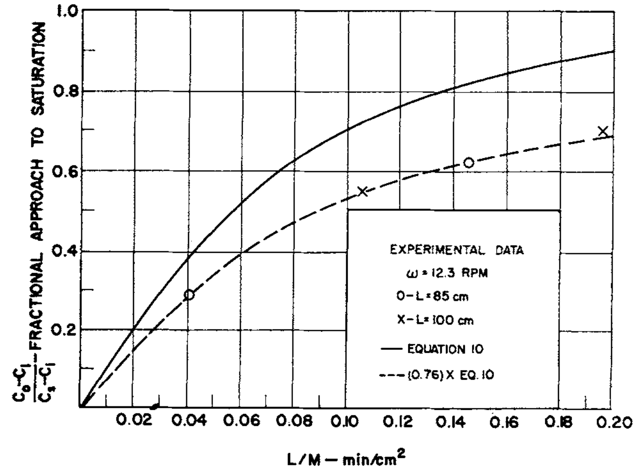

Fig. 7. Influence of the ratio *L/M* on the approach to saturation.

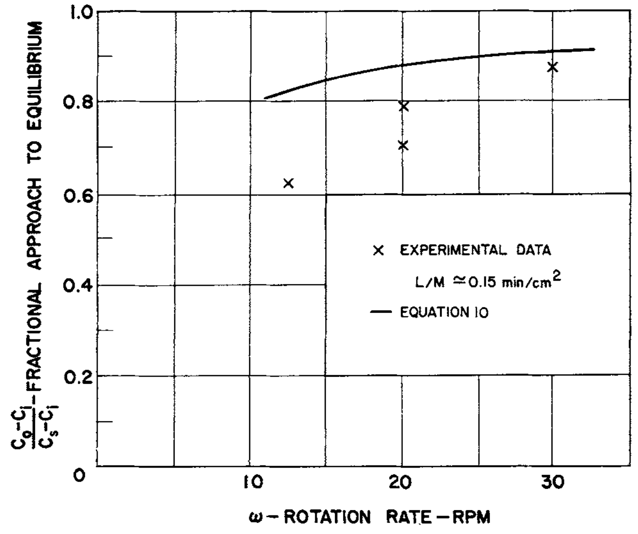

Fig. 8. Effect of rotation rate on the fractional approach to equilibrium.

Rotation Rate

As mentioned in the section on theory, rotation rate ω appears in both of the dimensionless groups a and G and therefore the influence of this variable is more difficult to anticipate. Data were obtained under conditions of essentially constant values of the ratio of L/M at various rotation rates (runs 6, 7, 8, and 12) and these data are presented in Figure 8. The theory provides a good qualitative description of the influence of rotation rate as illustrated by the solid line in Figure 8, but in all cases the theory predicts a closer approach to equilibrium than was actually observed. Note also that the discrepancy between the predicted and observed results is quite small at the highest rotation rate and increases as the rotation rate decreases.

USE OF OXYGENATOR FOR PERFUSION OF A DOG

The preliminary experiments which were carried out with the carbon dioxide–water system indicated quite reasonable agreement between theory and experiment, especially at relatively rapid rotation rates. Therefore the theoretical relation which was developed for the horizontal rotating cylinder [Equation (10)] was applied together with data from the literature and measured operating variables to predict the performance of the horizontal rotating cylinder as a blood oxygenator (*6*). These calculations indicated that the contactor should provide enough mass transfer of oxygen to supply the normal requirements of a dog at rest. Therefore, it was decided to use the blood oxygenator for an in vivo experiment.

A healthy stray dog that was scheduled for gassing was obtained for the test. It has been found that it is much more difficult to support a dog with a blood oxygenator than it is to support a human being, and therefore dogs are quite commonly used for tests for oxygenators.

Arrangements were made with a surgical team at Mercy Hospital, Oklahoma City, and the blood oxygenator and a mechanical heart were connected to the dog. The rate of oxygen transfer was not sufficient to support the dog as was evidenced by continual darkening of the dog's blood until the dog failed. The one experiment which was carried out with blood and oxygen in the horizontal rotating cylinder thereby indicates a significant difference between theoretically predicted rates of oxygen uptake and those which are actually obtained.

A number of reasons for this discrepancy could be advanced but the most probable source of error in the analysis is the result of poor mixing in the blood pool. Even in the experiments with the carbon dioxide–water system, poor mixing seems to be in evidence when the cylinder is rotated at low rates. Use of normal blood in the oxygenator would be expected to provide less mixing in the pool than experienced with water. This factor was probably accentuated in this case as it was noted by the nurse responsible for making the oxygen analyses that the viscosity of the blood of this particular dog was unusually high.

Another factor that may be of importance is the rate of equilibration between the red cell and the plasma. This rate was assumed to be instantaneous in order to obtain an analytical solution in closed form and this assumption should be checked further by additional theoretical and experimental investigations.

CONCLUSION

The horizontal rotating cylinder oxygenator appears to offer several advantages in that the device requires no debubbling equipment, the blood is subjected to very little shear in passing through the device, and the blood holdup is very low. Present results indicate that lack of mixing in the pool may be the cause of the low rates of oxygen transfer obtained when applied to the oxygenation of blood, but it is felt that proper design and operation can overcome this deficiency. It appears that additional development and laboratory experiments are required.

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NOTATION

- a = dimensionless group defined by Equation (9), $= (D/mH)(T/\omega Rt^2)$
- b = infinite series given by Equation (8)
- $C(x, y)$ = concentration of total available oxygen in blood at position x, y in the film, vol. %
- $C(z)$ = concentration of total available oxygen in blood in the pool of the oxygenator at location z, vol. %
- C_i = concentration of total available oxygen in blood entering the oxygenator, vol. %
- C_o = concentration of total available oxygen in blood leaving the oxygenator, vol. %
- C_s = concentration of total available oxygen in blood when saturated at surrounding conditions, vol. %
- $C^*(z)$ = concentration of total available oxygen in blood in the film as it reenters the pool, vol. %
- $C'(x, y)$ = concentration of oxygen in solution in the blood plasma at position x, y in the film, vol. %
- D = ordinary diffusion coefficient
- G = dimensionless group defined by Equation (11); $= (\omega RtL/M)$
- H = Henry's law constant
- j = intercept of straight line approximation to curved lines in Figure 3 [Equation (6)]
- L = total length of oxygenator including sump
- M = volumetric flow rate of liquid through horizontal rotating cylinder
- m = slope of straight line approximation to curved lines in Figure 3 [Equation (6)]
- N = slope of straight line through origin as approximation to curved lines in Figure 3 [Equation (5)]
- n = index used in Equation (8)
- p_{O_2} = partial pressure of oxygen
- R = radius of cylindrical section of smaller diameter
- T = length of film in x direction ($\approx$ circumference)
- t = uniform film thickness
- x = pseudo Cartesian coordinate tangent to cylinder in direction of rotation of cylinder
- y = pseudo Cartesian coordinate perpendicular to wall of cylinder
- z = coordinate parallel to axis of rotating cylinder
- ω = cylinder rotation rate
- π = numerical constant 3.1459 . . .

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Forced Convection Mass Transfer:

Part III. Increased Mass Transfer from a Flat Plate Caused by the Wake from Cylinders Located Near the Edge of the Boundary Layer

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Enhanced rates of mass transfer in the wake region behind detached cylindrical turbulence promoters were investigated with the use of the naphthalene sublimation technique. The maximum increase in the average rate of mass transfer through laminar boundary layers was over 170%. The remarkable feature of these results was that the enhanced rates of mass transfer persisted for over one hundred and thirty cylinder diameters downstream from the cylinder generating the wake. The observed effect was not only strongly dependent on the free stream velocity and the location of the cylinders relative to the mass transfer surface, but there were marked differences observed between the rate of mass transfer in the wake region behind one and behind two cylinders. These results resemble a "tuning phenomenon" and are believed to be due to Tollmein-Schlichting instabilities and premature transition.

In the second paper of this series (*1*) it was shown that the local and average rates of forced convection through laminar boundary layers on a flat plate were markedly increased by locating small cylinders near the outer edge of the boundary layer. The local rate of forced convection was strongly peaked directly beneath each cylinder; the magnitude of the effect depended upon the free stream velocity, the spacing between cylinders, and the gap between the cylinders and the plate. Under optimum conditions, local values of the rate of forced convection were increased as much as 240%, while the average values were increased by over 90%. The principal factors responsible for the observed peaking of the local rate of forced convection beneath cylinders were believed to be an increase in the velocity gradient at the surface and the interaction of the cylinder wake with the fluid in the boundary layer (*1*).

The wake interaction is undoubtedly quite complicated. In the first place, for the cylinder diameter and velocities covered in the second paper (*1*), the Reynolds number of the cylinder was above the threshold value for vortex street formation (*2*), provided the cylinder was located far from a surface. In other studies (*3*) it was shown that when multiple cylinders were located one behind the other in the plane of flow of the free stream, the vortex streets interacted in a very complex fashion. Contraction, expansion, cancellation, and coalescence of vortices were observed to occur for different values of cylinder separation and Reynolds number. Little information is available on the further complication introduced because of the proximity of the cylinders to a surface.

In addition to its intrinsic interest the isolation of wake interaction effects on the local rate of forced convection is a prerequisite to the understanding of the complicated dependence of the magnitude of the increase in the local rate of forced convection beneath cylinders on the velocity and/or the length Reynolds number (*1*).

WAKE INTERACTIONS

Many instances of interactions of wakes and secondary flows with fluid in the boundary layer are known. In general, the result of the interactions appears as excitations of inherent instabilities [such as Tollmein-Schlichting waves (*5*)], as secondary flows, or as other modifications of the boundary-layer velocity profile which may cause an increase in the rate of forced convection. However, little information is available on the magnitude of the increase in the rate of forced convection which would be caused by these flow modifications.

Tollmein-Schlichting waves and eventually premature boundary-layer transition have been shown to be excited by four or six 0.006-in. diameter wires located a few inches apart attached to the surface of a convex plate with a 20-ft. radius of curvature (*6*) and by multiple wires 0.002- to 0.1875-in. diameter in contact with the surface of a flat plate (*7, 8*). Hot wire investigations showed (*6*) that maximum amplification of the Tollmein-Schlichting